Precision tests of the Standard Model with leptonic and semileptonic kaon decays

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This paper presents the analysis of leptonic and semileptonic kaon decays data done by the FlaviaNet Kaon Working group, as described in [1]. Data include all recent results by BNL-E865, KLOE, KTeV, ISTRA+, and NA48. Experimental results are critically reviewed and combined, taking into account theoretical (both analytical and numerical) constraints on the semileptonic kaon form factors. We report on a very accurate determination of V_{us} as well as on many other tests of the SM which can be performed with leptonic and semileptonic K decays.

1. Introduction

In the Standard Model, SM, transition rates of semileptonic processes such as $d^i \to u^j \ell\nu$, with d^i (u^j) being a generic down (up) quark, can be computed with high accuracy in terms of the Fermi coupling G_F and the elements V_{ji} of the Cabibbo-Kobayashi Maskawa (CKM) matrix. Measurements of the transition rates provide therefore precise determinations of the fundamental SM couplings.

A detailed analysis of semileptonic decays offers also the possibility to set stringent constraints on new physics scenarios. While within the SM all $d^i \to u^j \ell\nu$ transitions are ruled by the same CKM coupling V_{ji} (satisfying the unitarity condition $\sum_k |V_{ik}|^2 = 1$) and G_F is the same coupling appearing in the muon decay, this is not necessarily true beyond the SM. Setting bounds on the violations of CKM unitarity, violations of lepton universality, and deviations from the $V-A$ structure, allows us to put significant constraints on various new-physics scenarios (or eventually find evidences of new physics).

In the case of leptonic and semileptonic K decays these tests are particularly significant given the large amount of data recently collected by several experiments: BNL-E865, KLOE, KTeV, ISTRA+, and NA48. The analysis of these data provides precise determination of fundamental SM couplings, sets stringent SM test almost free from hadronic uncertainties, and finally can discriminate between new physics scenarios. The high statistical precision of measurements and the detailed information on kinematical distributions have pushed a substantial progress on the theory side, in particular the theoretical error on hadronic form factors has been reduced at the 1% level.

The paper is organized as follows. First in Sec. 2 we present fits to world data on the leading branching ratios and lifetimes, for K_L, K_S, and $K^\pm$ mesons. Sec. 3 summarizes the status of the knowledge of form factor slopes from $K_{\ell 3}$ decays. The physics results obtained are described in Sec. 4, in particular the measurement of $|V_{us} f_+(0)|$. Finally, to the special role of $\Gamma(K^\pm_{e2})/\Gamma(K^\pm_{\mu 2})$ ratio is devoted the Sec. 5.

2. Experimental data: BRs and lifetime

Numerous measurements of the principal kaon BRs, or of various ratios of these BRs, have been published recently. For the purposes of evaluating $|V_{us} f_+(0)|$, these data can be used in a PDG-like fit to the BRs and lifetime, so all such measurements are interesting. A detailed description to the fit procedure and the references of all experimental input used can be found in Ref. [1].

For K_L the results are given in table 1, while table 2 gives the results for $K^\pm$. For the K_S, the fit is dominated by the KLOE measurements of $BR(K_S \to \pi e\nu)$ and of $BR(\pi^+\pi^-)/BR(\pi^0\pi^0)$.

*WWW access at www.lnf.infn.it/wg/vus/; for sake of completeness and brevity for all references we refer to the Note written by the FlaviaNet Kaon WG [1].

Parameter	Value	S
BR(K_{e3})	0.4056(7)	1.1
BR($K_{\mu 3}$)	0.2705(7)	1.1
BR($3\pi^0$)	0.1951(9)	1.2
BR($\pi^+\pi^-\pi^0$)	0.1254(6)	1.1
BR($\pi^+\pi^-$)	$1.997(7) \times 10^{-3}$	1.1
BR($2\pi^0$)	$8.64(4) \times 10^{-4}$	1.3
BR($\gamma\gamma$)	$5.47(4) \times 10^{-4}$	1.1
τ_L	51.17(20) ns	1.1

Table 1
Results of fit to K_L BRs and lifetime.

Parameter	Value	S
BR($K_{\mu 2}$)	63.57(11)%	1.1
BR($\pi\pi^0$)	20.64(8)%	1.1
BR($\pi\pi\pi$)	5.595(31)%	1.0
BR(K_{e3})	5.078(26)%	1.2
BR($K_{\mu 3}$)	3.365(27)%	1.7
BR($\pi\pi^0\pi^0$)	1.750(26)%	1.1
$\tau_\pm$	12.384(19) ns	1.7

Table 2
Results of fit to $K^\pm$ BRs and lifetime.

These, together with the constraint that the K_S BRs must add to unity, and the assumption of universal lepton couplings, completely determine the K_S leading BRs In particular, BR($K_S \to \pi e \nu$) $= 7.046(91) \times 10^{-4}$. For τ_{K_S} we use 0.8958×10^{-10} s, where this is the non-CPT constrained fit value from the PDG.

3. Experimental data: $K_{\ell 3}$ form factors

The hadronic $K \to \pi$ matrix element of the vector current is described by two form factors (FFs), $f_+(t)$ and $f_0(t)$. By construction, $f_0(0) = f_+(0)$. In order to compute the phase space integrals we need experimental or theoretical inputs about the t-dependence of FF. In principle, Chiral Perturbation Theory (ChPT) and Lattice QCD are useful tools to set theoretical constraints. However, in practice the t-dependence of the FFs at present is better determined by measurements and by combining measurements and dispersion relations. Many approaches have been used, and all have been described in detail in [1]. Here we list only the averages of quadratic fit results for K_{e3} and $K_{\mu 3}$ slopes (Table 3) used to determine $|V_{us}|f_+(0)$.

	K_L and K^-
Measurements	16
χ^2/ndf	54/13 (7×10^{-7})
$\lambda'_+ \times 10^3$	24.9 ± 1.1 ($S = 1.4$)
$\lambda''_+ \times 10^3$	1.6 ± 0.5 ($S = 1.3$)
$\lambda_0 \times 10^3$	13.4 ± 1.2 ($S = 1.9$)
$\rho(\lambda'_+, \lambda''_+)$	-0.94
$\rho(\lambda'_+, \lambda_0)$	$+0.33$
$\rho(\lambda''_+, \lambda_0)$	-0.44
$I(K^0_{e3})$	0.15457(29)
$I(K^\pm_{e3})$	0.15892(30)
$I(K^0_{\mu 3})$	0.10212(31)
$I(K^\pm_{\mu 3})$	0.10507(32)
$\rho(I_{e3}, I_{\mu 3})$	$+0.63$

Table 3
Averages of quadratic fit results for K_{e3} and $K_{\mu 3}$ slopes.

4. Physics results

4.1. Determination of $|V_{us}|f_+(0)$ and $|V_{us}|/|V_{ud}|f_K/f_\pi$

The value of $|V_{us}|f_+(0)$ has been determined from the decay rate of kaon semileptonic decays (see [1] for the detailed decomposition). using the world average values reported in previous sections for lifetimes, branching ratios and phase space integrals. The results are shown in figure 1 for $K_L \to \pi e \nu$, $K_L \to \pi \mu \nu$, $K_S \to \pi e \nu$, $K^\pm \to \pi e \nu$, $K^\pm \to \pi \mu \nu$, and for the combination. The average, $|V_{us}|f_+(0) = 0.21664(48)$, has an uncertainty of about of 0.2%. The results from the five modes are in good agreement, the fit probability is 58%. In particular, comparing the values of $|V_{us}|f_+(0)$ obtained from $K^0_{\ell 3}$ and $K^\pm_{\ell 3}$ we obtain a value of the SU(2) breaking correction $\delta^K_{SU(2)_{exp.}} = 2.9(4)\%$ in agreement with the

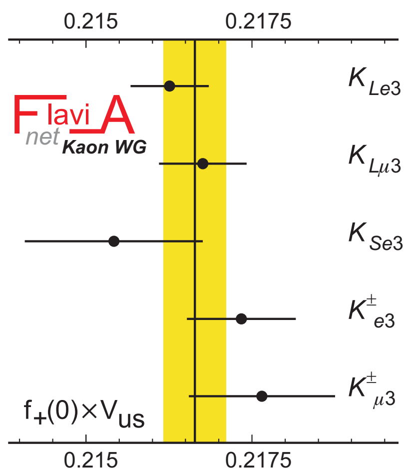

Figure 1. Display of $|V_{us}|f_+(0)$ for all channels.

CHPT calculation $\delta^K_{SU(2)} = 2.36(22)\%$. Moreover, recent analyzes on the so-called violations of Dashen's theorem in the kaon electromagnetic mass splitting point to $\delta^K_{SU(2)}$ values of about 3%.

The test of Lepton Flavor Universality (LFU) between K_{e3} and $K_{\mu3}$ modes constraints a possible anomalous lepton-flavor dependence in the leading weak vector current. It can therefore be compared to similar tests in τ decays, but is different from the LFU tests in the helicity-suppressed modes π_{l2} and K_{l2}. The results on the parameter $r_{\mu e} = R^{\rm Exp}_{K_{\mu3}/K_{e3}}/R^{\rm SM}_{K_{\mu3}/K_{e3}}$ is $r_{\mu e} = 1.0043 \pm 0.0052$, in excellent agreement with lepton universality. With a precision of 0.5% the test in K_{l3} decays has now reached the sensitivity of other determinations: $r_{\mu e}(\tau) = 1.0005 \pm 0.0041$ and $r_{\mu e}(\pi) = 1.0042 \pm 0.0033$ [2]

An independent determination of V_{us} is obtained from $K_{\ell 2}$ decays. The most important mode is $K^+ \to \mu^+\nu$, which has been recently updated by KLOE reaching a relative uncertainty of about 0.3%. Hadronic uncertainties are minimized considering the ratio $\Gamma(K^+ \to \mu^+\nu)/\Gamma(\pi^+ \to \mu^+\nu)$. Using the world average values of $\mathrm{BR}(K^\pm \to \mu^\pm\nu)$ and of $\tau^\pm$ given in Section 2 and the value of $\Gamma(\pi^\pm \to \mu^\pm\nu) = 38.408(7)\ \mu s^{-1}$ from [2] we obtain: $|V_{us}|/|V_{ud}|f_K/f_\pi = 0.2760 \pm 0.0006$.

4.2. Theoretical estimates of $f_+(0)$ and f_K/f_π

The main obstacle in transforming these highly precise determinations of $|V_{us}|f_+(0)$ and $|V_{us}|/|V_{ud}|f_K/f_\pi$ into a determination of $|V_{us}|$ at the per-mil level are the theoretical uncertainties on the hadronic parameters $f_+(0)$ and f_K/f_π. This hadronic quantity cannot be computed in perturbative QCD, but it is highly constrained by $SU(3)$ and chiral symmetry. In the chiral limit and, more generally, in the $SU(3)$ limit ($m_u = m_d = m_s$) the conservation of the vector current implies $f_+(0)$=1. Expanding around the chiral limit in powers of light quark masses we can write $f_+(0) = 1 + f_2 + f_4 + \ldots$ where f_2 and f_4 are the NLO and NNLO corrections in ChPT. The Ademollo–Gatto theorem implies that $(f_+(0) - 1)$ is at least of second order in the breaking of $SU(3)$ This in turn implies that f_2 is free from the uncertainties of the $\mathcal{O}(p^4)$ counterterms in ChPT, and it can be computed with high accuracy: $f_2 = -0.023$. The difficulties in estimating $f_+(0)$ begin with f_4 or at $\mathcal{O}(p^6)$ in the chiral expansion. Several analytical approaches to determine f_4 have been attempted over the years, essentially confirming the original estimate by Leutwyler and Roos. The benefit of these new results, obtained using more sophisticated techniques, lies in the fact that a better control over the systematic uncertainties of the calculation has been obtained. However, the size of the error is still around or above 1%, which is not comparable to the 0.2% accuracy which has been reached for $|V_{us}|f_+(0)$.

Recent progress in lattice QCD gives us more optimism in the reduction of the error on $f_+(0)$ below the 1% level. Most of the currently available lattice QCD results have been obtained with relatively heavy pions and the chiral extrapolation represents the dominant source of uncertainty. There is a general trend of lattice QCD results to be slightly lower than analytical approaches. An important step in the reduction of the error associated to the chiral extrapolation has been recently made by the UKQCD-RBC collaboration. Their preliminary result $f_+(0) =$

0.964(5) is obtained from the unquenched study with $N_F = 2+1$ flavors, with an action that has good chiral properties on the lattice even at finite lattice spacing (domain-wall quarks). They also reached pions masses ($\geq$ 330 MeV) much lighter than that used in previous studies of $f_+(0)$. The overall error is estimated to be 0.5%, which is very encouraging.

In contrast to the semileptonic vector form factor, the pseudoscalar decay constants are not protected by the Ademollo–Gatto theorem and receive corrections linear in the quark masses. Expanding f_K/f_π in power of quark masses, in analogy to $f_+(0)$, $f_K/f_\pi = 1 + r_2 + \ldots$ one finds that the $\mathcal{O}(p^4)$ contribution r_2 is already affected by local contributions and cannot be unambiguously predicted in ChPT. As a result, in the determination of f_K/f_π lattice QCD has essentially no competition from purely analytical approaches. The present overall accuracy is about 1%. The novelty are the new lattice results with $N_F = 2+1$ dynamical quarks and pions as light as 280 MeV, obtained by using the so-called staggered quarks. These analyzes cover a broad range of lattice spacings (i.e. a=0.06 and 0.15 fm) and is performed on sufficiently large physical volumes ($m_\pi L \geq 5.0$). It should be stressed, however, that the sensitivity of f_K/f_π to lighter pions is larger than in the computation of $f_+(0)$ and that chiral extrapolations are far more demanding in this case. In the following analysis we will use as reference value the MILC-HPQCD result $f_K/f_\pi = 1.189(7)$.

4.3. Test of CKM unitarity

To determine $|V_{us}|$ and $|V_{ud}|$ we use the value $|V_{us}|f_+(0) = 0.2166(5)$, the result $|V_{us}|/|V_{ud}|f_K/f_\pi = 0.2760(6)$, $f_+(0) = 0.964(5)$, and $f_K/f_\pi = 1.189(7)$. From the above we find: $|V_{us}| = 0.2246 \pm 0.0012$ from $K_{\ell 3}$ only, and $|V_{us}|/|V_{ud}| = 0.2321 \pm 0.0015$ from $K_{\ell 2}$ only. These determinations can be used in a fit together with the the recent evaluation of V_{ud} from $0^+ \to 0^+$ nuclear beta decays: $|V_{ud}|$=0.97418±0.00026. This global fit gives $V_{ud} = 0.97417(26)$ and $V_{us} = 0.2253(9)$, with $\chi^2/\mathrm{ndf} = 0.65/1$ (42%). This result does not make use of CKM unitarity. If the unitarity constraint is included, the fit gives $V_{us} = 0.2255(7)$ and $\chi^2/\mathrm{ndf} = 0.80/2$ (67%). Both results are illustrated in Fig. 2. The test of CKM unitarity can be also interpreted as a test of universality of the lepton and quark gauge couplings. Using the results of the fit (without imposing unitarity) we obtain: $G_{\mathrm{CKM}} \equiv G_\mu \left[|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\right]^{1/2} = (1.1662 \pm 0.0004) \times 10^{-5}\ \mathrm{GeV}^{-2}$, in perfect agreement with the value obtained from the measurement of the muon lifetime: $G_\mu = (1.166371 \pm 0.000007) \times 10^{-5}\ \mathrm{GeV}^{-2}$. The current accuracy of the lepton-quark universality sets important constraints on model building beyond the SM. For example, the presence of a Z' would affect the relation between G_{CKM} and G_μ. In case of a Z' from $SO(10)$ grand unification theories we obtain $m_{Z'} > 700$ GeV at 95% CL, to be compared with the $m_{Z'} > 720$ GeV bound set through the direct collider searches [2]. In a similar way, the unitarity constraint also provides useful bounds in various supersymmetry-breaking scenarios.

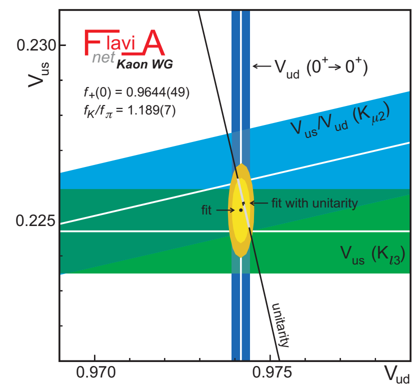

Figure 2. Results of fits to $|V_{ud}|$, $|V_{ux}|$, and $|V_{us}|/|V_{ud}|$.

4.4. $K_{\ell 2}$ sensitivity to new physics

A particularly interesting test is the comparison of the $|V_{us}|$ value extracted from the helicity-suppressed $K_{\ell 2}$ decays with respect to the value extracted from the helicity-allowed $K_{\ell 3}$ modes. To reduce theoretical uncertainties from f_K and electromagnetic corrections in $K_{\ell 2}$, we exploit the

ratio $BR(K_{\ell 2})/BR(\pi_{\ell 2})$ and we study the quantity

$$R_{l23} = \left| \frac{V_{us}(K_{\ell 2})}{V_{us}(K_{\ell 3})} \frac{V_{ud}(0^+ \to 0^+)}{V_{ud}(\pi_{\ell 2})} \right| .$$

Within the SM, $R_{l23} = 1$, while deviation from 1 can be induced by non-vanishing scalar- or right-handed currents. Notice that in R_{l23} the hadronic uncertainties enter through $(f_K/f_\pi)/f_+(0)$. In the case of effect of scalar currents due to a charged Higgs, the unitarity relation between $|V_{ud}|$ extracted from $0^+ \to 0^+$ nuclear beta decays and $|V_{us}|$ extracted from $K_{\ell 3}$ remains valid as soon as form factors are experimentally determined. This constrain together with the experimental information of $\log C^{MSSM}$ can be used in the global fit to improve the accuracy of the determination of R_{l23}, which in this scenario turns to be $R_{l23}|^{\rm exp}_{\rm scalar} = 1.004 \pm 0.007$. Here $(f_K/f_\pi)/f_+(0)$ has been fixed from lattice. This ratio is the key quantity to be improved in order to reduce present uncertainty on R_{l23}. This measurement of R_{l23} can be used to set bounds on the charged Higgs mass and $\tan\beta$. Figure 3 shows the excluded region at 95% CL in the M_H–$\tan\beta$ plane. The measurement of BR($B \to \tau\nu$) can be also used to set a similar bound in the M_H–$\tan\beta$ plane. While $B \to \tau\nu$ can exclude quite an extensive region of this plane, there is an uncovered region in the exclusion corresponding to a destructive interference between the charged-Higgs and the SM amplitude. This region is fully covered by the $K \to \mu\nu$ result.

4.5. A test of lattice calculation

The vector and scalar form factors $f_{+,0}(t)$ are analytic functions in the complex t–plane, except for a cut along the positive real axis, starting at the first physical threshold $t_{\rm th} = (m_K + m_\pi)^2$, where they develop discontinuities. They are real for $t < t_{\rm th}$. Cauchy's theorem implies that $f_{+,0}(t)$ can be written as a dispersive integral along the physical cut where all possible on-shell intermediate states contribute to its imaginary part. A number of subtractions is needed to make the integral convergent. Particularly appealing is an improved dispersion relation recently proposed where two subtractions

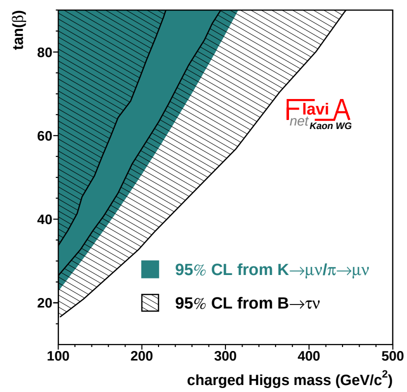

Figure 3. Excluded region in the charged Higgs mass-$\tan\beta$ plane. The region excluded by $B \to \tau\nu$ is also indicated.

are performed at $t = 0$ (where by definition, $\tilde{f}_0(0) \equiv 1$) and at the so-called Callan-Treiman point $t_{CT} \equiv (m_K^2 - m_\pi^2)$. Since the Callan-Treiman relation fixes the value of scalar form factor at t_{CT} to the ratio $(f_K/f_\pi)/f_+(0)$, the dispersive parametrization for the scalar form factor allows to transform the available measurements of the scalar form factor into a precise information on $(f_K/f_\pi)/f_+(0)$, completely independent of the lattice estimates. Figure 4 shows the values for $f_+(0)$ determined from the scalar form factor slope measurements obtained using a dispersive parametrization and the Callan-Treiman relation, and $f_K/f_\pi = 1.189(7)$. from result on the FF slope using the dispersive parameterization The value of $f_+(0) = 0.964(5)$ from UKQCD/RBC is also shown.

5. The special role of of $\Gamma(K_{e2})/\Gamma(K_{\mu 2})$

The ratio $R_K = \Gamma(K_{\mu 2})/\Gamma(K_{e2})$ can be precisely calculated within the Standard Model. Neglecting radiative corrections, it is given by $R_K^{(0)} = \frac{m_e^2}{m_\mu^2} \frac{(m_K^2 - m_e^2)^2}{(m_K^2 - m_\mu^2)^2} = 2.569 \times 10^{-5}$, and reflects the strong helicity suppression of the electron channel. Radiative corrections have been computed with effective theories, yielding the final SM prediction $R_K^{\rm SM} = R_K^{(0)}(1 + \delta R_K^{\rm rad.corr.}) =$

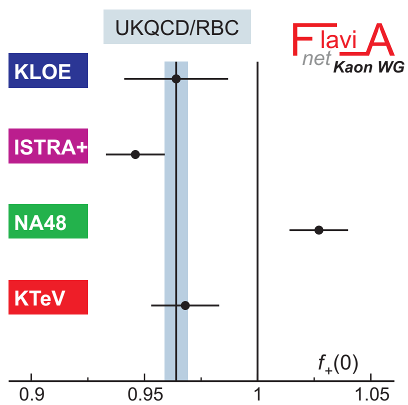

Figure 4. Values for $f_+(0)$ determined from the scalar form factor slope using the Callan-Treiman relation and $f_K/f_\pi = 1.189(7)$.

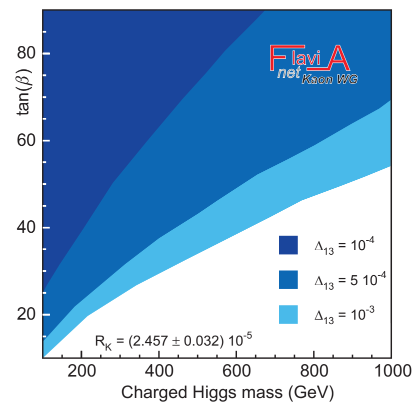

Figure 5. Exclusion limits at 95% CL on $\tan\beta$ and the charged Higgs mass $M_{H^\pm}$ from $|V_{us}|_{K\ell 2}/|V_{us}|_{K\ell 3}$ for different values of Δ_{13}.

$2.569 \times 10^{-5} \times (0.9622 \pm 0.0004) = (2.477 \pm 0.001) \times 10^{-5}$. Because of the helicity suppression within then SM, the K_{e2} amplitude is a prominent candidate for possible sizable contributions from physics beyond the SM. Moreover, when normalizing to the $K_{\mu 2}$ rate, we obtain an extremely precise prediction of the K_{e2} width within the SM. In order to be visible in the $K_{e2}/K_{\mu 2}$ ratio, the new physics must violate lepton flavor universality.

Recently it has been pointed out that in a supersymmetric framework sizable violations of lepton universality can be expected in K_{l2} decays. At the tree level, lepton flavor violating terms are forbidden in the MSSM. However, these appear at the one-loop level, where an effective $H^+ l \nu_\tau$ Yukawa interaction is generated. The non-SM contribution to R_K can be written as $R_K^{\rm LFV} \approx R_K^{\rm SM}\left[1+\left(\frac{m_K^4}{M_{H^\pm}^4}\right)\left(\frac{m_\tau^2}{M_e^2}\right)|\Delta_{13}|^2\tan^6\beta\right]$, where Δ_{13}, the lepton flavor violating coupling , being generated at the loop level, could reach values of $\mathcal{O}(10^{-3})$. For moderately large $\tan\beta$ values, this contribution may therefore enhance R_K by up to a few percent.

Experimental knowledge of $K_{e2}/K_{\mu 2}$ has been poor so far. The current world average of $R_K = \mathrm{BR}(K_{e2})/\mathrm{BR}(K_{\mu 2}) = (2.45 \pm 0.11) \times 10^{-5}$ dates back to three experiments of the 1970s [2] and has a precision of about 5%. Three new preliminary measurements were reported by NA48/2 and KLOE (see [1] for details). Both, the KLOE and the NA48/2 measurements are inclusive with respect to final state radiation contribution due to bremsstrahlung. Combining these new results with the current PDG value yields a current world average of $R_K = (2.457 \pm 0.032) \times 10^{-5}$, with a relative error of 1.3%, a factor three more precise than the previous world average. This value is in very good agreement with the SM expectation and gives strong constraints for $\tan\beta$ and $M_{H^\pm}$, as shown in Fig. 5. For values of $\Delta_{13} \approx 5 \times 10^{-4}$ and $\tan\beta > 50$ the charged Higgs masses is pushed above 1000 GeV/c^2 at 95% CL.

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